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RESEARCH MEMORANDUM

PRESSURE DISTRIBUTION AND AERODYNAMIC COEFFICIENTS ASSOCIATED WITH HEAT ADDITION TO SUPERSONIC AIR STREAM ADJACENT TO TWO-DIMENSIONAL SUPERSONIC WING

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

The modifications in the pressure distributions and the aerodynamic coefficients associated with addition of heat to the two-dimensional supersonic inviscid flow field adjacent to the lower surface of a 5-percent-thickness symmetrical circular-arc wing are presented in this report. The pressure distributions are obtained by the use of a graphical method which gives the two-dimensional supersonic inviscid flow field obtained with moderate heat addition. The variation is given of the lift-drag ratio and of the aerodynamic coefficients of lift, drag, and moment with free-stream Mach number, angle of attack, and parameters defining extent and amount of heat addition. The six graphical solutions used in this study included Mach numbers of 3.0 and 5.0 and angles of attack of 0° and 2°.

A moderate addition of heat in the manner described produced a significant increase in the lift coefficient, the moment coefficient, and the lift-drag ratio. For the wing shape and the angles of attack considered, this addition of heat generated some thrust which appeared as a slight decrease in the drag coefficient for the graphical solutions obtained.

An equation is presented for computing the heat addition required to obtain a given increase in total temperature across the heated stream tube.

INTRODUCTION

The effect of moderate heat addition on the flow and state parameters of the flow fields of various aerodynamic and propulsive bodies has been considered for some time (references 1 to 3). In reference 1

is presented a theory for the one-dimensional flow of a steady compressible fluid flow in ducts with heat addition. The problem of a small rate of heat addition in the vicinity of a circular cylinder in a subsonic flow field which is uniform at infinity is considered in reference 2. The addition of heat to the external air flow around projectiles, the so-called external ram jet, is considered in reference 3. The theory developed by Hicks (reference 2) is at the present time, however, limited to the treatment of special subsonic compressible inviscid flow problems. By means of the graphical method presented in reference 4, the two-dimensional flow fields with moderate heat addition to the flow about given aerodynamic or propulsive bodies for supersonic inviscid flows can be constructed. 2375

This report represents an appraisal of the modifications in pressure distribution and the aerodynamic coefficients associated with heat addition to the flow underneath a two-dimensional wing in supersonic flight. Essentially, it gives a preliminary survey made at the NACA Lewis laboratory of the effect of some of the important variables involved. Included among these variables are flight Mach number, stagnation-temperature ratio across the heated portion of the flow, chordwise distribution of heat added to the flow, and wing angle of attack. All pressure distribution computations are made for a 5-percent-thickness symmetrical two-dimensional supersonic wing (no sweep back) whose upper and lower surfaces are circular arcs. The results obtained apply as a first approximation to two-dimensional sharp-edged wings having a 5-percent thickness but a different profile. No consideration is given to the problem of the means by which heat addition to a supersonic flow may be accomplished.

METHOD AND SCOPE OF COMPUTATIONS

For all cases considered herein, the pressure distributions associated with the addition of heat to the flow were obtained by the graphical method for mapping a two-dimensional supersonic flow with heat addition described in reference 4. This graphical method is valid for heat addition to a supersonic fluid flow in which heat conduction does not occur. The wing section employed for this study is illustrated schematically in figure 1. The symmetrical 5-percent-thickness airfoil has surfaces which are circular arcs with radius of curvature of 5.0125 chords. The symbols to be used in the following presentation are listed in appendix A. In figure 1 the heat addition region A beneath the wing is downstream of the leading-edge shock wave and is considered to be adjacent to the wing surface. In order to apply the method of graphical solution, the heat addition region A is considered to be divided into chordwise intervals in each interval of which a constant fractional change in total temperature $\Delta T/T$ occurs. In the region A the lines of constant stagnation temperature are approximately normal to the local streamlines (reference 4).

The principal geometric variables employed herein are defined in appendix A and are illustrated in figure 1. This study is made for the stagnation temperature ratios of 1.243 and 1.126 and the free-stream Mach numbers of 3.0 and 5.0. The combination of variables covered herein appear in table I.

From the pressure distributions around the wing obtained by the graphical construction of the flow the wing lift, drag, and moments about the quarter-chord point were obtained from the customary expressions defining these terms.

RESULTS AND DISCUSSION

On the basis of the limited number of cases computed, general conclusions cannot be drawn from the results obtained. A sufficient number of cases were chosen, however, to illustrate the order of change in lift and drag produced by change in angle of attack, free-stream Mach number, and the principal characteristics of the heat-addition process.

Pressure Distribution

In figure 2 are shown the pressure distributions about the wing without and with heat addition. The examples shown in this figure correspond to and are in the same order as those given in table I. Because an idealized frictionless two-dimensional flow is assumed, no interaction between the flow below and above the wing occurs. For this reason, in the present report the pressure distributions over the top surface of the wing are not influenced by the heat addition to the flow below the wing. The principal effect of this heat addition is the marked increase in local static pressure along the lower surface of the wing downstream of the heat-addition starting point as compared with the case of no heat addition.

Aerodynamic Coefficients

Effect of varying free-stream Mach number. - The variation of the aerodynamic coefficients with the free-stream Mach number when the other parameters being considered in this study are held constant is plotted in figure 3. The aerodynamic coefficients in the figure were calculated from the pressure distributions of examples 1 and 2, which are plotted in figures 2(a) and 2(b), respectively. The coefficients of lift and pitching moment about the quarter-chord point are given in figures 3(a) and 3(b), respectively. The ratio of the lift coefficient with heat addition to the lift coefficient without heat addition is 1.67 at M_0

equal to 3.0 and 2.00 at M_0 equal to 5.0. The effect of increasing the free-stream Mach number on the quarter-chord moment coefficient is more marked, the ratio of the quarter-chord moment coefficient with heat addition to its value without heat addition being 2.47 and 4.23 for M_0 equal to 3.0 and 5.0, respectively.

From the plot of the variation in pressure-drag coefficient with free-stream Mach number M_0 (fig. 3(c)) it is evident that only a modest thrust is generated with the heat addition to the flow as reflected in the small reduction in pressure drag associated with heat addition. The thrust generated by the increased wing-surface pressure associated with heat addition to the flow depends on the shape of the lower wing surface and its angle of inclination to the free-stream direction. Some gain in thrust over that realized in these examples can be obtained by providing lower wing-surface elements, inclined to give a forward pressure-force component in the direction of flight over a large section of the lower surface appreciating a pressure rise from heat additions to the flow. It should be noted that the values of drag used in figures 3 to 7 have been calculated from the pressure distributions and do not include values of viscous drag. Although this pressure-drag reduction amounts to only 4.8 percent at M_0 equal to 3.0 and 10 percent at M_0 equal to 5.0, the corresponding marked increase in lift provides the favorable lift-drag ratio L/D plotted in figure 3(d). In this figure is shown an L/D with heat addition equal to 1.67 and 2.24 times the L/D without heat addition at M_0 equal to 3.0 and 5.0, respectively.

Effect of varying angle of attack. - The manner in which wing angle of attack modifies the effect that heat addition to the flow has on the aerodynamic coefficients as evaluated from the pressure distribution given for examples 2 and 6 (figs. 2(b) and 2(f)) is illustrated in figure 4. The examples considered differ only in the angle of attack, which is 2° for example 2 and 0° for example 6. Because the value of h_i/c was kept the same for example 6 as for example 2 the value of the weight flow of heated air for example 6 at zero angle of attack is 91.2 percent of that for example 2 at an angle of attack of 2°. It is evident that the increase in lift coefficient realized from the addition of heat is almost constant at a value of approximately 0.028 for zero and 2° angles of attack (fig. 4(a)). A similar conclusion can be drawn regarding the effect of heat addition on the moment coefficient, the change in moment coefficient obtained with heat addition being -0.0127 at zero angle of attack and -0.0139 at an angle of attack of 2° (fig. 4(b)).

As shown in figure 4(c) the pressure-drag-coefficient reduction associated with heat addition falls from 0.0011 at zero angle of attack to 0.0003 at 2° angle of attack. The smaller effectiveness of the heat addition in reducing the pressure-drag coefficient for the angle of attack of 2° stems from the fact that at this angle of attack the drag

components of the pressure forces on the lower surface of the wing are reduced because of the orientation of the wing with respect to the free-stream direction. For the wing shape considered, the corresponding L/D values shown in figure 4(d) reflect this effect in that at zero angle of attack the increase in L/D due to heat addition is 7.7 and at an angle of attack of 2°, 5.0.

Effect of varying h_i/c. - An appraisal of the variation of the 2375 aerodynamic coefficients with the initial extent of the heat-addition region A (fig. 1) normal to the wing surface h_i/c is presented in figure 5. The quantities plotted in figures 5(a) and 5(d) are obtained from the pressure distributions of examples 2 and 3, which are shown in figures 2(b) and 2(c), respectively. The variation of the lift coefficient with the initial extent of heat addition normal to the surface shows that the lift coefficient increase due to heat addition for $h_i/c = 0.035$ (example 3) is 0.0179 and the lift coefficient increase for $h_i/c = 0.07$ (example 2) is 0.0302 (fig. 5(a)). The increase in lift coefficient provided by doubling the initial width of the heated stream tube to 0.07 chord and, hence, doubling the heat addition to the flow is about 20 percent less than twice the lift coefficient increase for the heated stream tube of 0.035 chord. A similar result is obtained for the moment coefficient (fig. 5(b)). The reduced modification of the wing pressure distribution provided by the heat added to the flow at stream tubes having the larger separation from the lower surface of the wing is due in part to the smaller portion of the wing chord influenced. The first portion of the wing that appreciates the modifying effects produced by the heat addition in these stream tubes lies along the Mach lines from the point of initiation of heat addition. The intersection of the Mach lines with the lower wing surface is downstream of the chordwise location of the initiation of the heat addition. The drag coefficient decreases slightly as the initial extent of heat addition normal to the surface is increased (fig. 5(c)). Up to an initial heated stream-tube width of 0.07 chord the drag coefficient decreases linearly with an increase in h_i/c. However, because the variation of the drag coefficient with h_i/c is slight, the variation of L/D with the initial extent of heat addition normal to the surface given in figure 5(d) is quite similar to that of the lift coefficient. For the heated zone 0.035 chord in width, the increase in L/D due to heat addition is 3.2 as compared with 2.3 for the added increase in L/D provided by broadening the heated region to 0.07 chord.

Effect of varying chordwise extent of heat addition keeping constant stagnation-temperature ratio. - The effect on the aerodynamic parameters of changing the chordwise extent of the heat-addition region is obtained from the results of examples 2 and 5 (fig. 6). The change in the chordwise extent of the heat-addition region while keeping constant the stagnation-temperature ratio across the heated region is obtained by

holding constant the initial width of the heat-addition region h_i/c and the value of the chordwise position where the heat addition is initiated x_i/c and by changing the value of the chordwise position of the heat addition termination x_f/c and the representative chordwise rate of heat addition $\frac{dT}{dx}$. In example 5 the total heat addition occurs over a chordwise distance of 0.334 chord and in example 2, over a distance of 0.644 chord with the same value of x_i'/c and the same total heat addition taking place in both cases. From figures 6(a) and 6(b) it is evident that the more restricted heat-addition region of 0.334 chord is associated with greater effectiveness of the heat addition in increasing the lift and the moment coefficients. When the heat-addition region extends for 0.644 chord the increased lift and moment coefficients are 78 percent and 97 percent, respectively, of the values corresponding to the heat-addition region having a chordwise length of 0.334 chord. The greater effectiveness of the shorter heat-addition regions stems largely from the fact that the pressure rise associated with heat addition is effective over an increased portion of the wing chord when all the heat addition is completed over an increased distance forward of the wing trailing edge.

The drag-coefficient values (fig. 6(c)) show that for the case of heat addition over 0.334 chord the drag increases slightly over the case for no heat addition, whereas the region of 0.644 chord provides a reduced drag coefficient amounting to 96 percent of the drag associated with no heat addition. Because the lift and drag coefficients decrease with chordwise extension of the heat-addition region over the range considered in this study at approximately the same rate, the lift-drag ratio is constant with change of the length of the heat-addition region (fig. 6(d)).

Effect of varying stagnation-temperature ratio T_f/T_0 with constant chordwise heat-addition rate. - The wing pressure distributions corresponding to examples 2 and 4 (table I), given in figures 2(b) and 2(d), provide a first appraisal of how the aerodynamic coefficients are affected by varying the chordwise extent of the heat addition while holding constant the chordwise rate of heat addition. Thus, for both examples 2 and 4 the ratio of the final stagnation temperature to the initial stagnation temperature T_f/T_0 is a function only of the chordwise extent of the heat-addition region. The increase in lift coefficient provided by heat addition increases with an increase in stagnation-temperature ratio (fig. 7(a)). On the other hand, the slope of the curve showing the lift coefficient plotted against T_f/T_0 decreases with increasing values of T_f/T_0. Because the heat addition which occurs close to the wing trailing edge can modify the wing pressure distribution over a correspondingly small percentage of the chord, the decline in slope with increasing T_f/T_0 of the curve showing the lift coefficient plotted against T_f/T_0 can be expected for the mode of heat

addition considered. If the heat addition is provided by the uniform combustion of special fuels mixed with the air stream, the constant chordwise rate of heat addition is approximated. These results indicate the desirability of completing the combustion of the fuel over a short chordwise distance and, as shown by the preceding section, as far upstream along the chord as can be permitted. The moment coefficient rises almost linearly with T_f/T_0 in the limited range of T_f/T_0 explored in this study (fig. 7(b)).

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The drag coefficient plotted against T_f/T_0 (fig. 7(c)) indicates that no significant change in C_d occurs until the value of T_f/T_0 of 1.16 is exceeded. This characteristic variation of C_d with T_f/T_0 and the declining rate of increase of C_d with T_f/T_0 combine to give the lift-drag-ratio curve of figure 7(d), which is almost linear with T_f/T_0 up to a value of 1.246.

Heat-Energy Considerations

Definition and significance of heat-addition coefficient. - In addition to a discussion of the manner in which the pressure distribution and aerodynamic coefficients are affected by the heat addition to supersonic flow in a region adjacent to the lower surface of the wing, a presentation of the manner in which the heat energy requirements vary for the various examples considered in this report is pertinent. In appendix B is presented a derivation of the expression for the heat addition per unit time and unit span. This equation has the form

$$H = \left[c(t_0)^{1/2}p_0\right]\left[\left(\frac{g\gamma}{R}\right)^{1/2}c_p\left(1+\frac{\gamma-1}{2}N_0\right)^{1/2}\left(\frac{T_f}{T_0}-1\right)\sum_{0}^{h_i/c}\left\{\frac{p_i}{p_0}\left[N_i\left(1+\frac{\gamma-1}{2}N_i\right)\right]^{1/2}\Delta\left(\frac{y_i}{c}\right)\right\}\right] \qquad (1)$$

where the symbols are defined in appendix A.

It is convenient to consider the expression for the quantity H (equation (1)) as composed of two factors. The first factor is $c(t_0)^{1/2}p_0$ which depends only on the chord of the wing and the flight altitude. The second factor is

$$C_H = \left(\frac{g\gamma}{R}\right)^{1/2}c_p\left(1+\frac{\gamma-1}{2}N_0\right)^{1/2}\left(\frac{T_f}{T_0}-1\right)\sum_{0}^{h_i/c}\left\{\frac{p_i}{p_0}\left[N_i\left(1+\frac{\gamma-1}{2}N_i\right)\right]^{1/2}\Delta\left(\frac{y_i}{c}\right)\right\}$$

the terms of which are provided by the graphical solution of the flow field for a known wing profile, free-stream Mach number, angle of attack, and program of heat addition. The quantity C_H can be considered as a heat-addition coefficient which has the dimensions, British thermal units per second per pound per degree Rankine to the one-half power. The convenience of this coefficient arises from the fact that the graphical solution for the flow around a given wing at a stated angle of attack and prescribed manner of heat addition is a function of the free-stream Mach number only. Flight altitude influences the terms p_0 and $(t_0)^{1/2}$ only. The coefficients of heat addition for the six examples considered are listed in table I.

The following illustrative calculation is given as an indication of the usefulness of the coefficient of heat addition. From the graphical solution of example 2 (table I), which has $M_0 = 3.0$, $\alpha = 2^\circ$, $x_i/c = 0.356$, $x_f/c = 1.000$, $h_i/c \doteq 0.07$, $T_f/T_0 = 1.243$, and $C_H = 0.0387$, the value of the heat addition H in terms of British thermal units per second per unit span is

$$H = \left[0.0387\right] \left[c(t_0)^{1/2} p_0\right]$$

For a wing chord of 10 feet and a flight altitude of 100,000 feet where, from the NACA standard atmosphere (reference 5), $p_0 = 22.4$ pounds per square foot and $t_0 = 392.4^\circ$ R, the value of H is

$$H = \left[0.0387\right] \left[(10)(\sqrt{392.4})(22.4)\right] = 172 \text{ Btu/(sec)(unit span)}$$

Using the NACA standard atmosphere at an altitude of 30,000 feet gives

$$H = \left[0.0387\right] \left[(10)(\sqrt{412.1})(628.5)\right] = 4930 \text{ Btu/(sec)(unit span)}$$

Illustrative application. - In order to illustrate those circumstances under which a ram-jet-powered supersonic airplane can appreciate a reduction in fuel consumption by utilizing the device of heat addition to the wing flow, a simplified analysis is presented for a hypothetical airplane in appendix C. The ram-jet-propelled airplane is assumed to fly at 30,000 feet in level steady flight. The fuselage drag is assumed to represent one-fourth of the total airplane drag, and the total wing drag per unit span is taken to be the same whether or not heat addition to the wing flow is employed. For the wing studied herein the pressure

drag is not influenced markedly by the wing-flow heat addition. The assumption regarding the wing drag per unit span is therefore satisfactory as a first order approximation. Three-dimensional wing effects are not considered in the analysis. A thrust specific fuel consumption of 3 pounds of fuel per hour per pound of thrust is charged to the ram jet.

The results of the analysis of appendix C are summarized in table I. These results indicate that if the airplane lift-drag ratio is less than a value from 3 to 4, depending on the quantity and mode of heat addition to the wing flow, some fuel economy results from heat addition to the wing flow.

In this illustrative example the reduced wing area permitted by the added lift provided by the heated wing flow is responsible for drag reduction which results in fuel economy. The airplane with and without wing-flow heat addition is considered to fly at a Mach number of 3.0 and an angle of attack of 2°. When heat addition to the flow is to be used intermittently for short periods of added performance, the wing span is fixed by the unheated flow condition. Airplane drag reduction with heat addition to the flow results from the decreased drag associated with flight at reduced angle of attack permitted by increased lift associated with heat addition.

CONCLUDING REMARKS

From a study of six graphical solutions which include free-stream Mach numbers of 3.0 and 5.0, angles of attack of 0° and 2°, and various parameters defining extent and amount of heat addition, the following remarks may be made:

1. The addition of heat to the two-dimensional supersonic flow adjacent to the lower surface of a 5-percent-thickness circular-arc symmetrical airfoil produced a significant increase in the lift-drag ratio of the wing, amounting to 1.67 times the unheated value at a free-stream Mach number of 3.0 and an angle of attack of 2°. Similar trends were noted for the lift-drag ratio and the moment coefficient.

2. In general, for the examples at an angle of attack of 2° and free-stream Mach number of 3.0 the addition of heat to the region underneath the circular-arc symmetrical wing resulted in slightly decreased pressure-drag coefficients. This improvement in pressure drag may be lost at higher angles of attack. Because the effect of heat addition to the flow on pressure drag is influenced to a marked degree by wing shape and mode of heat distribution to the flow, general remarks on this subject cannot be made without further study.

3. A preliminary analysis comparing the fuel consumptions of a supersonic ram jet indicated that at a free-stream Mach number of 3.0 and for airplane lift-drag ratios less than 4, some fuel saving was achieved by utilizing the added lift provided by heat addition to the wing flow. Under other circumstances this device may represent a useful employment of waste engine heat or provide a means of obtaining added airplane speed for short periods of time.

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APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	region where the heat addition is occurring
c	chord of airfoil, ft
C_d	coefficient of pressure drag
C_H	coefficient of heat addition, $C_H = \dfrac{H}{c\left(t_0\right)^{1/2} p_0}, \dfrac{\text{Btu}}{(\text{sec})(\text{lb})(^{\circ}\text{R})^{1/2}}$
C_l	coefficient of lift
$\left(C_M\right)_{c/4}$	coefficient of moment about quarter-chord point
c_p	specific heat at constant pressure, Btu/(lb)($^{\circ}$F)
c_v	specific heat at constant volume, Btu/(lb)($^{\circ}$F)
D	wing pressure drag, lb
g	acceleration due to gravity, ft/sec^2
H	time rate of heat addition per unit span, Btu/(sec)(ft)
h_i	initial extent of heat-addition region as measured from the surface and approximately along the normal to the wing surface at that point, ft
h_0	displacement of limiting streamline of heat addition traced upstream through shock wave, measured from leading edge of wing in a direction normal to free-stream velocity, ft
L	wing lift, lb
M	Mach number
N	Mach number squared

p	static pressure, lb/sq ft
R	gas constant, ft-lb/(lb)($^{\circ}$R)
T	total temperature, $^{\circ}$R
t	static temperature, $^{\circ}$R
V	velocity, ft/sec
w	weight flow per unit span subjected to heat addition, lb/(sec)(ft)
x	horizontal coordinate of airfoil surface, parallel to chord line, ft
y	distance normal to surface of airfoil measured outward from surface, ft
α	wing angle of attack
γ	ratio of specific heats, c_p/c_v
ρ	density, slugs/cu ft

Subscripts:

0	free-stream conditions
f	station where heat addition is terminated
i	station where heat addition is begun

APPENDIX B

HEAT-ADDITION EQUATION

The heat added to the flow is given by conventional relations as

$$H = wC_p(T_f - T_0) \tag{B1}$$

where w is the heated weight flow of air per unit span. If the width of the stream tube involved in the addition of heat (fig. 1) is known at the leading edge of the wing, the heated mass flow per unit span can be expressed as

$$w = \rho_0 g V_0 h_0 \tag{B2}$$

where h_0 (fig. 1) terminates on the limiting streamline of heat addition traced upstream through the shock wave and is measured from the leading edge of the wing in a direction normal to the free-stream velocity. When the shape of the body departs from plane elements as illustrated in figure 1, however, the streamlines at station i are not parallel and determination of h_0 is obtained with some work. The graphical solution for the flow, however, does give the variation of Mach number with displacement normal to the streamlines along station i and the heated air weight flow can be obtained directly in the form

$$w\left[\frac{\text{lbs}}{(\text{sec})(\text{unit span})}\right] = \sum_{0}^{h_i/c} \rho_i g V_i c \Delta\left(\frac{y_i}{c}\right) \tag{B3}$$

where the subscript i refers to conditions at the station where heat addition is begun and where the values of ρ_i and V_i are constant over each increment of $\Delta(y_i/c)$. By means of the equation of state of an ideal gas and the definition of Mach number, equation (B3) becomes

$$w = \sum_{0}^{h_i/c} \left(\frac{g\gamma}{R}\right)^{1/2} p_i (t_i)^{-1/2} N_i^{1/2} c\Delta\left(\frac{y_i}{c}\right) \tag{B4}$$

With the substitution of equation (B4) in equation (B1) the relation for the heat added given in terms of British thermal units per second per unit span is

$$H = c_p T_0 \left(\frac{T_f}{T_0} - 1\right) \sum_{0}^{h_i/c} \left[\left(\frac{g\gamma}{R}\right)^{1/2} p_i (t_i)^{-1/2} N_i^{\ 1/2} c \Delta \left(\frac{y_i}{c}\right)\right] \tag{B5}$$

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From the relations

$$T_0 = t_0 \left(1 + \frac{\gamma - 1}{2} N_0\right)$$

and

$$t_i^{\ -1/2} = t_0^{\ -1/2} \left(1 + \frac{\gamma - 1}{2} N_0\right)^{-1/2} \left(1 + \frac{\gamma - 1}{2} N_i\right)^{1/2}$$

equation (B5) becomes

$$H = \left[c \left(t_0\right)^{1/2} p_0\right] \left[\left(\frac{g\gamma}{R}\right)^{1/2} c_p \left(1 + \frac{\gamma - 1}{2} N_0\right)^{1/2} \left(\frac{T_f}{T_0} - 1\right) \sum_{0}^{h_i/c} \left\{\left(\frac{p_i}{p_0}\right) \left[N_i \left(1 + \frac{\gamma - 1}{2} N_i\right)\right]^{1/2} \Delta \left(\frac{y_i}{c}\right)\right\}\right] \tag{1}$$

APPENDIX C

CALCULATION OF ENERGY REQUIREMENTS FOR SUPERSONIC AIRPLANE WITH AND WITHOUT HEAT ADDITION UNDERNEATH WING

Because of the marked gain in the ratio of lift to pressure drag provided by heat addition to the flow, it is desirable to evaluate under what circumstances an over-all advantage is obtained by dividing the airplane fuel between the engine air flow and the wing air flow. In order to make a simplified analysis of this problem, the supersonic airplane involved is considered to have wings of rectangular plan form and the two-dimensional-wing aerodynamic coefficients are assumed to apply over the entire wing span. The airplane is considered to be at an altitude of 30,000 feet in level steady flight at a Mach number of 3.0 with an angle of attack of 2°. No weight penalty is considered associated with the arrangements for adding heat to the wing flow. A thrust specific fuel consumption of 3 pounds of fuel per pound thrust per hour is charged to the ram-jet engine at this flight Mach number.

For the airplane without heat addition underneath the wing, the total fuel consumption in pounds of fuel per second may be expressed

$$\underset{\text{(no heat addition)}}{\text{Total fuel consumption rate}} = \frac{W_g}{(L/D)_{airpl}}\left(\frac{\text{thrust sfc}}{3600}\right)$$

$$= \frac{W_g}{(L/D)_{airpl}}\left(\frac{1}{1200}\right) \tag{C1}$$

where

W_g gross weight of airplane

$(L/D)_{airpl}$ lift-drag ratio of whole airplane

thrust sfc thrust specific fuel consumption, $\frac{\text{lb fuel}}{(\text{lb thrust})(\text{hr})}$

For the airplane with heat addition underneath the wing of rectangular plan form having span b and chord c the total fuel consumption in pounds of fuel per second may be expressed

$$\text{Total fuel consumption rate (with heat addition)} = \frac{W_g}{(L/D)^*_{airpl}}\left(\frac{1}{1200}\right) + \frac{H_{wing}}{Q_f} \qquad \text{(C2)}$$

where

Q_f heat value of fuel in British thermal units per pound of fuel

$(L/D)^*_{airpl}$ lift-drag ratio of whole airplane with heat addition underneath its wing 2375

The H_{wing} is the value in British thermal units per second of the heat addition needed to obtain the desired total-temperature distribution underneath a wing when the problem has been simplified by assuming that the two-dimensional-flow characteristics of the problem considered in the report apply to this rectangular wing of chord c and span b.

Because

$$H_{wing} = Hb = \left(C_H c \left(t_0\right)^{1/2} p_0\right) b \qquad \text{(C3)}$$

and

$$b = \frac{W_g}{C_l^* q_0 c} \qquad \text{(C4)}$$

where

C_H coefficient of heat addition defined in appendix A

b span, ft

c chord, ft

H time rate of heat addition per unit span, Btu/(sec)(ft)

t_0 free-stream static air temperature, $^{\circ}R$

p_0 free-stream static pressure, lb/sq ft

C_l^* coefficient of lift of wing with heat addition

q_0 free-stream dynamic pressure, lb/sq ft

from equations (C2), (C3), and (C4) the total fuel consumption for the airplane with heat addition underneath the wing becomes

$$\underset{\text{(with heat addition)}}{\text{Total fuel consumption rate}} = \frac{W_g}{(L/D)^*_{airpl}} \frac{1}{1200} + \frac{H}{Q_f} \frac{W_g}{C_l{}^* q_0 c} \qquad \text{(C5)}$$

The definition of the wing span b in terms of the lift coefficient with heat addition automatically adjusts the wing span to account for the increased lift associated with heat addition. In order to make a comparison of the performance of an airplane with and without heat addition it is convenient to take the ratio of the total airplane fuel consumption with heat addition to that without heat addition as follows:

$$\frac{\text{Total fuel consumption rate (with heat addition)}}{\text{Total fuel consumption rate (without heat addition)}} = \eta$$

$$= \frac{(L/D)_{airpl}}{(L/D)^*_{airpl}} + \frac{1200}{Q_f} \frac{H(L/D)_{airpl}}{C_l{}^* q_0 c} \qquad \text{(C6)}$$

In terms of fuel consumption the performance of an airplane configuration with heat addition underneath the wing is better than the performance of a similar airplane without heat addition underneath the wing whenever the fuel consumption ratio η given by equation (C6) is less than 1.

If it is assumed that all the airplane lift is obtained from the wings, the fuselage drag is the same for the cases with and without heat addition to the wing air flow, the total drag (viscous drag plus pressure drag) per unit span of the wing is also the same for both cases, and the body drag is one-fourth of the total airplane drag at $M_0 = 3.0$. Then

$$D_{airpl} = D_{wing} + D_{body} = \frac{W_g}{C_l q_0 c} D'_{wing} + 0.25\ D_{airpl}$$

$$= \frac{W_g}{0.75 C_l q_0 c} D'_{wing} \qquad \text{(C7)}$$

where

D'_{wing} drag of wing per unit span

$\frac{W_g}{C_l q_0 c}$ span of wing, b

and

$$D_{body} = 0.25\, D_{airpl} = \frac{0.25}{0.75} \frac{W_g}{C_l q_0 c} D'_{wing}$$

For the airplane with heat addition underneath the wing

$$D^*_{airpl} = D^*_{wing} + D_{body} = \frac{W_g}{C^*_l q_0 c} D'_{wing} + \frac{0.25}{0.75} \frac{W_g}{C_l q_0 c} D'_{wing}$$

$$= \frac{W_g}{C^*_l q_0 c} D'_{wing} + \frac{1}{3} \frac{W_g}{C_l q_0 c} D'_{wing} = \frac{W_g\, D'_{wing}}{q_0 c} \left[\frac{1}{C^*_l} + \frac{1}{3} \frac{1}{C_l}\right] \tag{C8}$$

Now because

$$L_{airpl} = L^*_{airpl} = W_g$$

the first term on the right side of equation (C6) is obtained as

$$\frac{(L/D)_{airpl}}{(L/D)^*_{airpl}} = \frac{D^*_{airpl}}{D_{airpl}} = \frac{3}{4}\left(\frac{C_l}{C^*_l} + \frac{1}{3}\right) \tag{C9}$$

and equation (C6) becomes

$$\eta = \frac{3}{4}\left(\frac{C_l}{C^*_l} + \frac{1}{3}\right) + \frac{1200}{Q_f} \frac{H(L/D)_{airpl}}{C^*_l q_0 c} \tag{C10}$$

If the heat value of the fuel Q_f is considered to be 18,700 British thermal units per pound and the relation for H from equation (1) is used in equation (C5),

$$\eta = \frac{3}{4}\left(\frac{C_l}{C^*_l} + \frac{1}{3}\right) + 0.207 \frac{C_H}{C^*_l} (L/D)_{airpl} \tag{C11}$$

where the values of q_0 at $M_0 = 3.0$ and t_0 at an altitude of 30,000 feet are used.

If the value of the lift-drag ratio of a typical supersonic airplane configuration without heat addition underneath the wing is assumed, then by use of equation (C6) the value of the total fuel consumption ratio η is obtained when the program of heat addition underneath the wing is specified.

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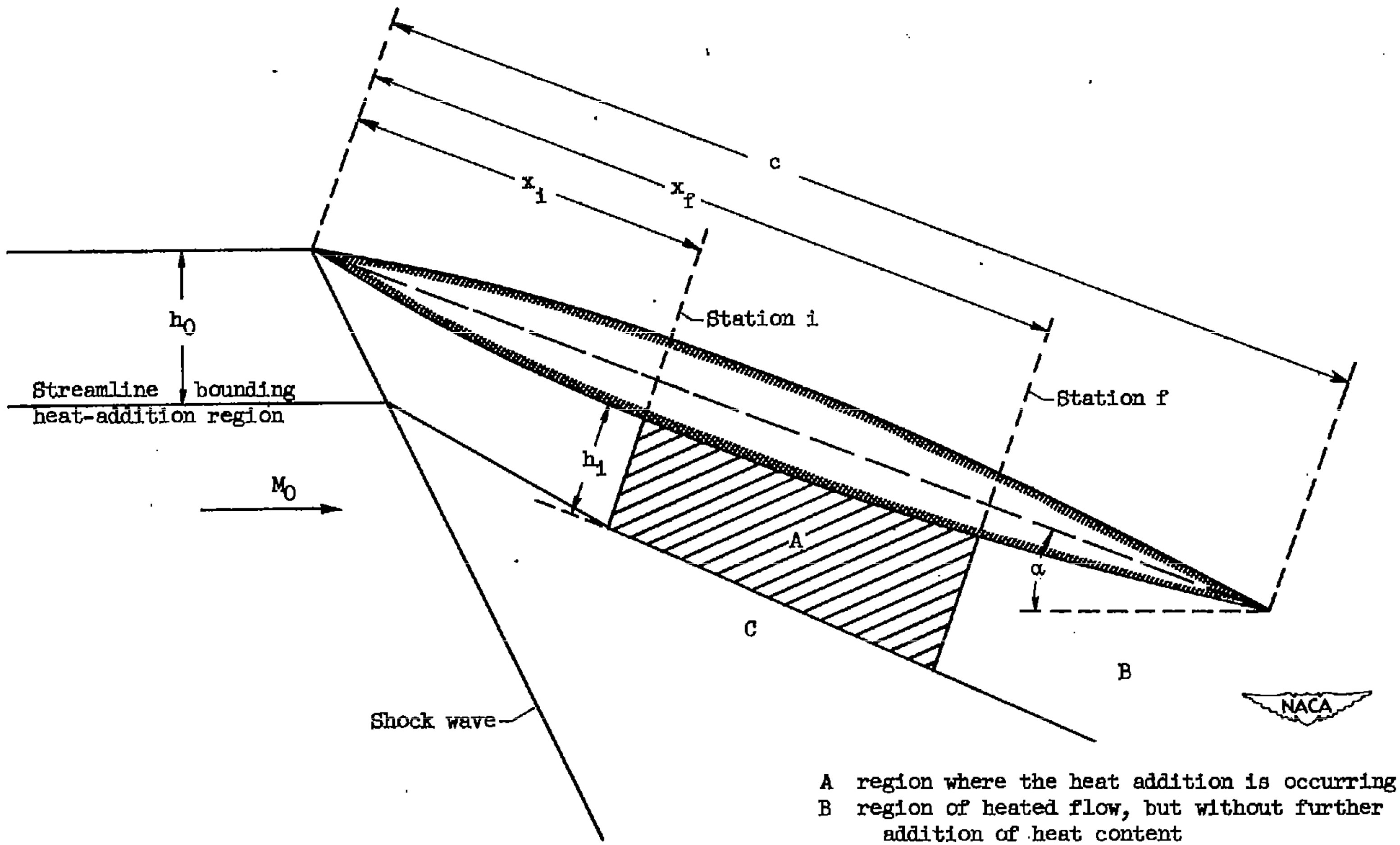
TABLE I - VALUES OF GEOMETRIC, FLOW, AND THERMODYNAMIC PARAMETERS

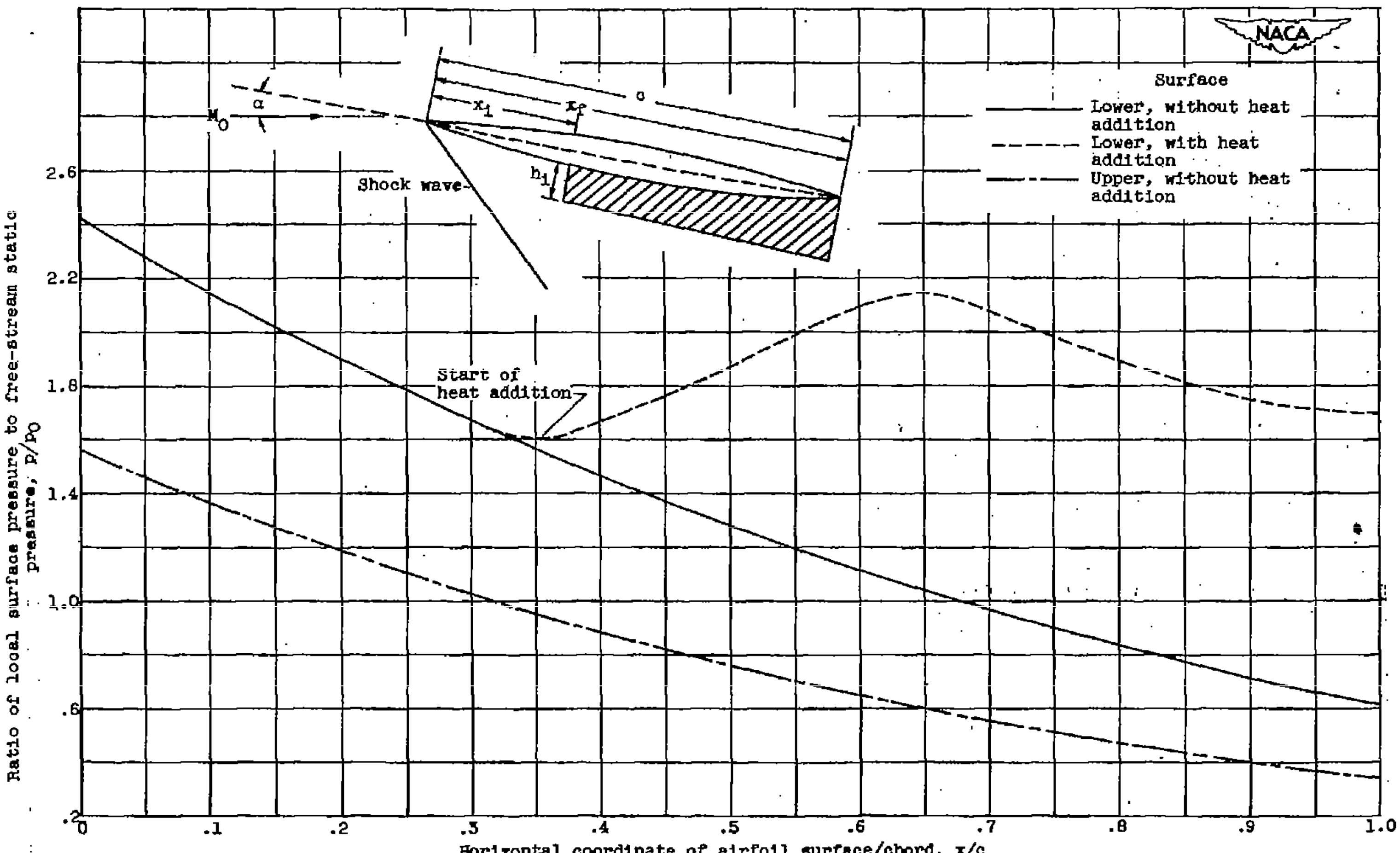
Example	Mach number M_0	Angle of attack α (deg)	$\frac{x_i}{c}$	$\frac{x_f}{c}$	$\frac{h_i}{c}$	$\frac{T_f}{T_0}$	Coefficient of heat addition C_H $\left(\frac{\text{Btu}}{(\text{sec})(\text{lb})(^{\circ}\text{R})^{1/2}}\right)$	Maximum lift-drag ratio of airplane for which fuel economy results from wing flow heat addition L/D (a)
1	5.00	2	0.356	1.000	0.07	1.243	0.1660	----
2	3.00	2	.356	1.000	.07	1.243	.0387	2.82
3	3.00	2	.356	1.000	.035	1.243	.0190	3.42
4	3.00	2	.356	.690	.07	1.126	.0201	3.76
5	3.00	2	.356	.690	.07	1.243	.0387	3.53
6	3.00	0	.356	1.000	.07	1.243	.0353	----

[a]Ratio of body drag to airplane drag, 1/4; altitude, 30,000 ft.



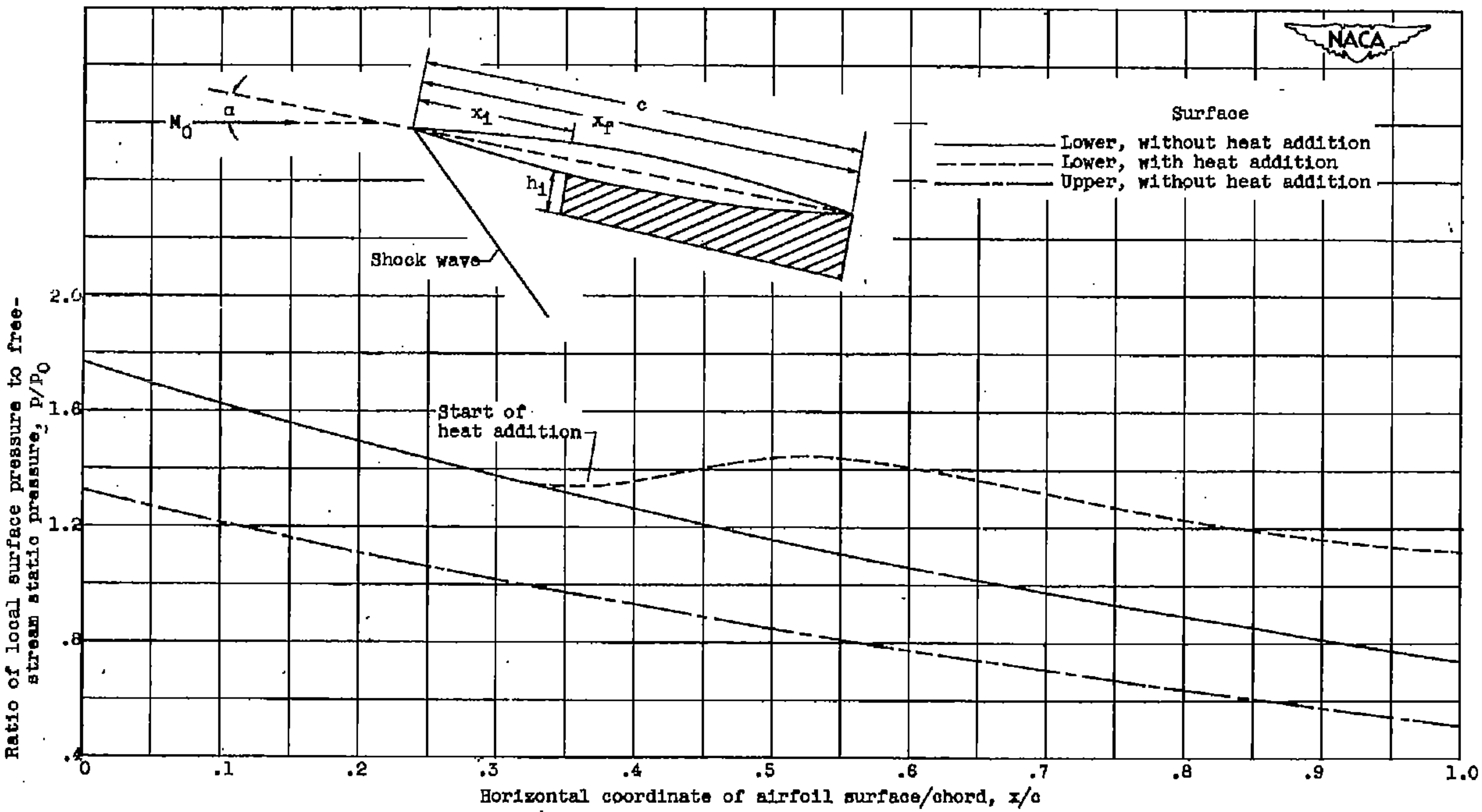
A region where the heat addition is occurring
B region of heated flow, but without further addition of heat content
C region of unheated flow

Figure 1. - Wing section indicating geometric parameters of flow.



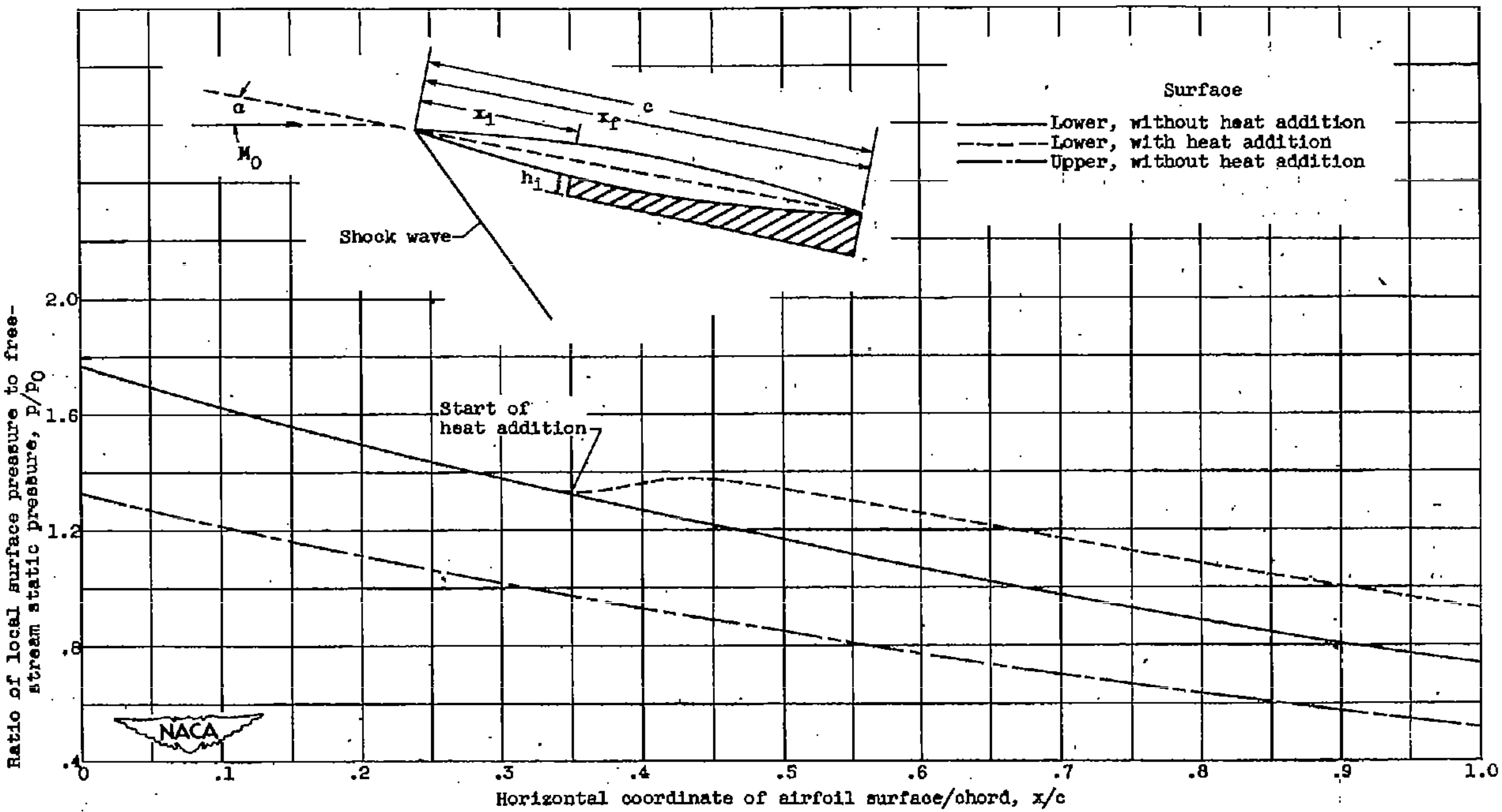
(a) Example 1: α, 2°; M_0, 5.00; T_f/T_0 = 1.243; x_1/c, 0.356; x_f/c, 1.000; h_1/c, 0.07.

Figure 2. - Surface pressure distributions obtained from graphical solution when heat is added in region adjacent to lower wing surface.



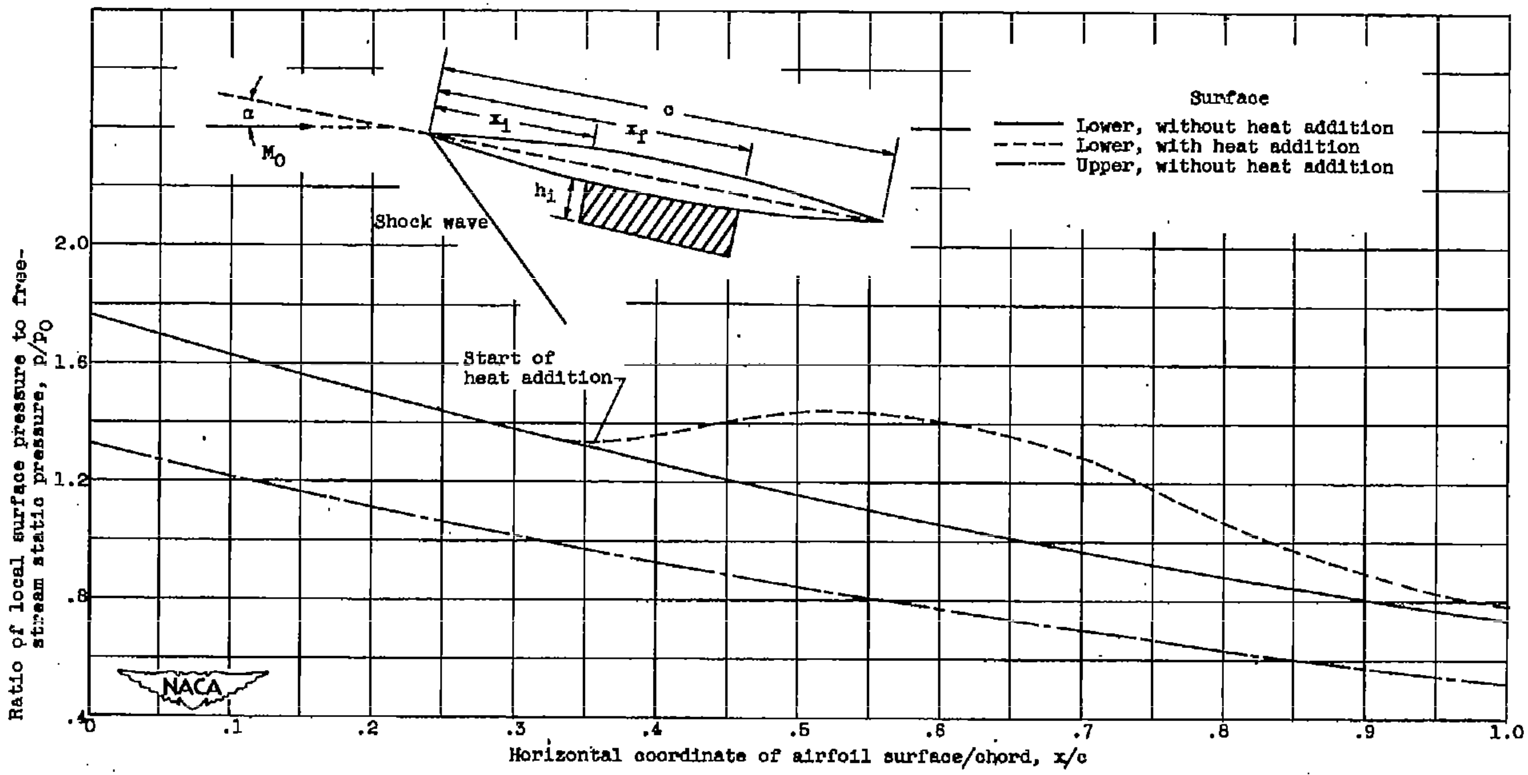
(b) Example 2: α, 2°; M_0, 3.00; T_f/T_0, 1.243; x_1/c, 0.356; x_f/c, 1.000; h_1/c, 0.07.

Figure 2. - Continued. Surface pressure distributions obtained from graphical solution when heat is added in region adjacent to lower wing surface.



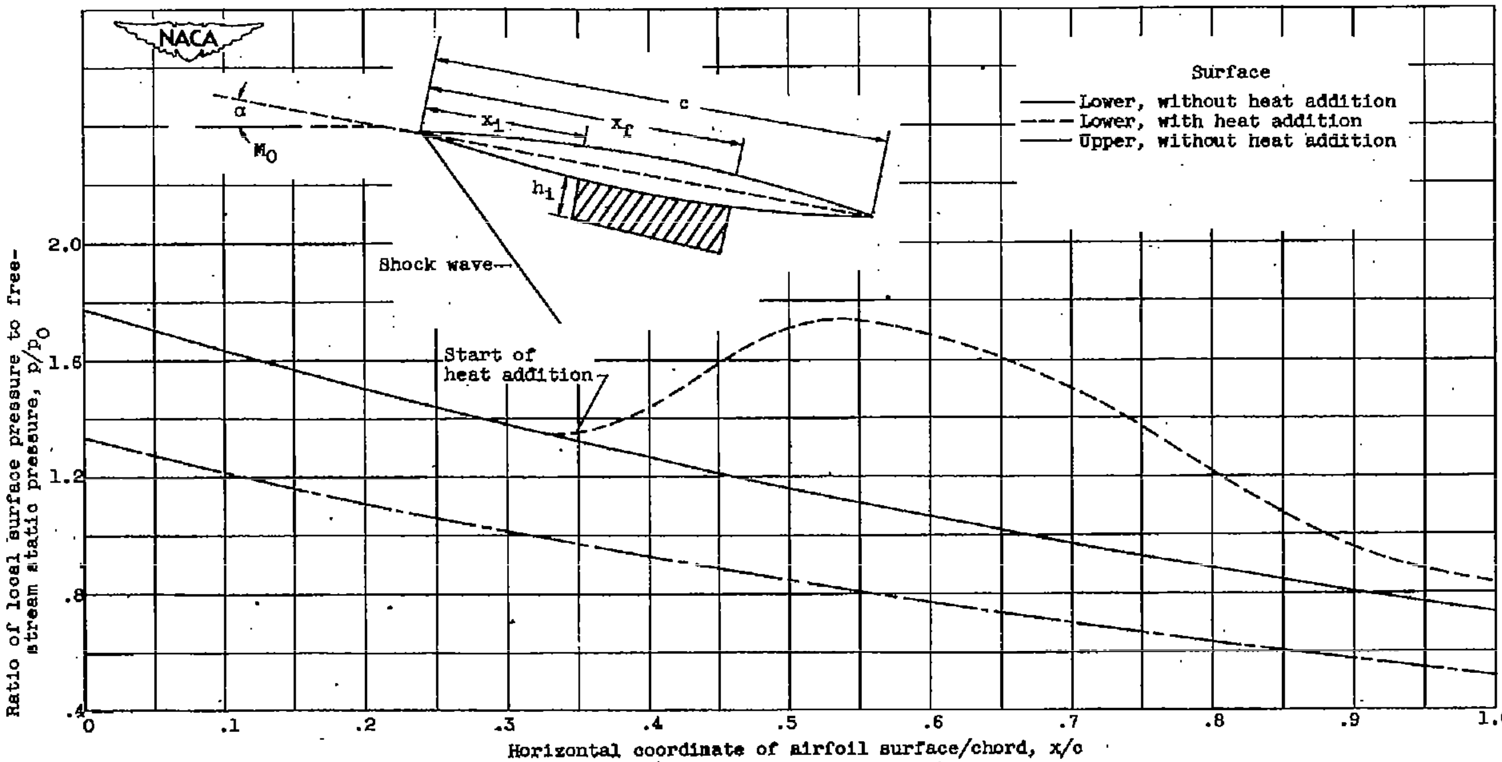
(c) Example 3: α, 2°; M_0, 3.00; T_f/T_0, 1.243; x_i/c, 0.356; x_f/c, 1.000; h_i/c, 0.035.

Figure 2. - Continued. Surface pressure distributions obtained from graphical solution when heat is added in region adjacent to lower wing surface.



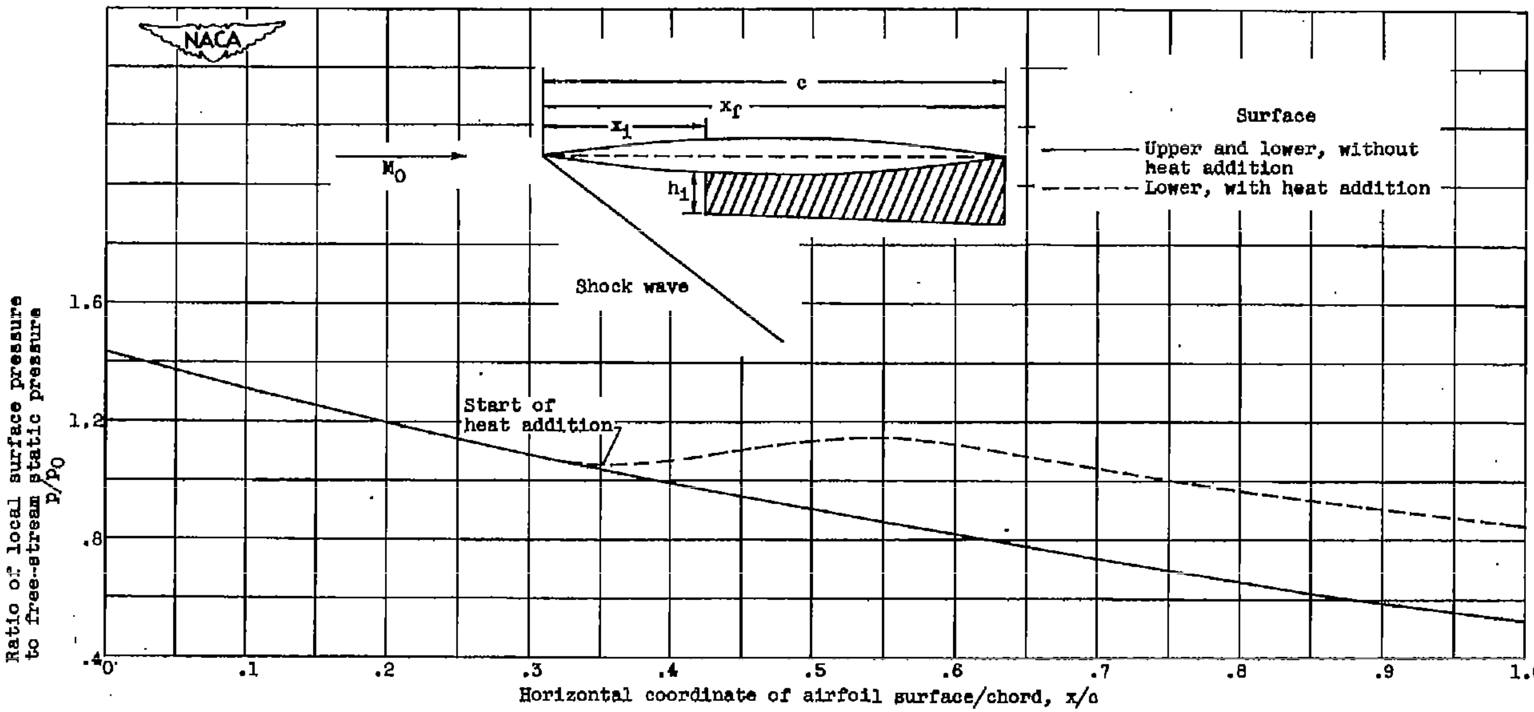
(d) Example 4: α, 2°; M_0, 3.00; T_f/T_0, 1.126; x_i/c, 0.356; x_f/c, 0.690; h_i/c, 0.07.

Figure 2. - Continued. Surface pressure distributions obtained from graphical solution when heat is added in region adjacent to lower wing surface.



(e) Example 5: α, 2°; M_0, 3.00; T_f/T_0, 1.243; x_1/c, 0.356; x_f/c, 0.690; h_1/c, 0.07.

Figure 2. - Continued. Surface pressure distributions obtained from graphical solution when heat is added in region adjacent to lower wing surface.



(f) Example 6: α, 0°; M_0, 3.00; T_f/T_0, 1.243; x_1/c, 0.356; x_f/c, 1.000; h_1/c, 0.07.

Figure 2. - Concluded. Surface pressure distributions obtained from graphical solution when heat is added in region adjacent to lower wing surface.

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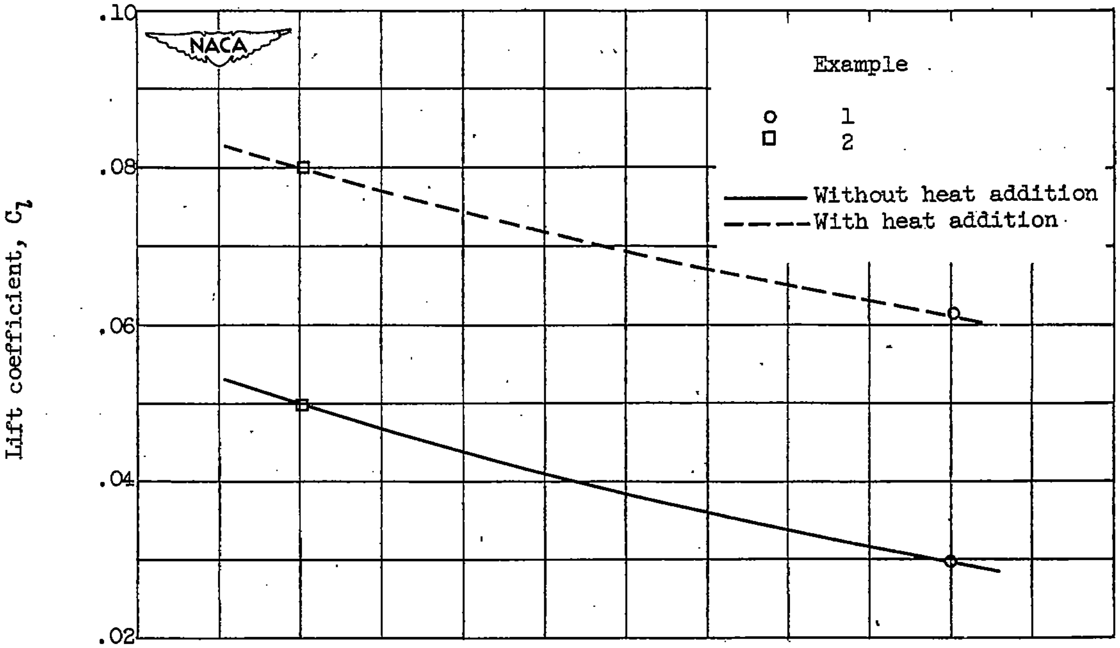
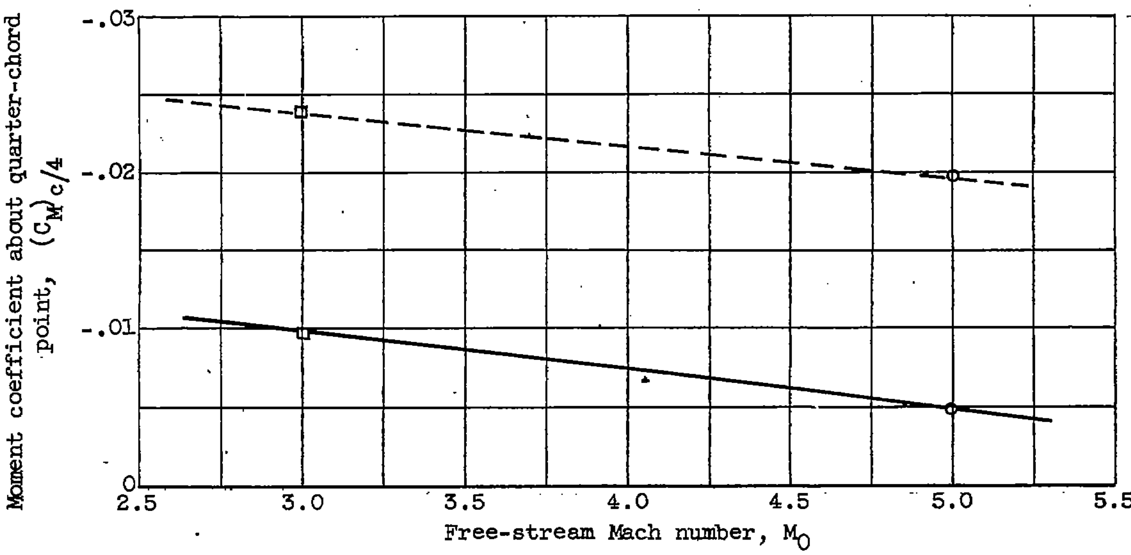


(a) Lift coefficient.



(b) Moment coefficient.

Figure 3. - Variation of aerodynamic coefficients with free-stream Mach number.

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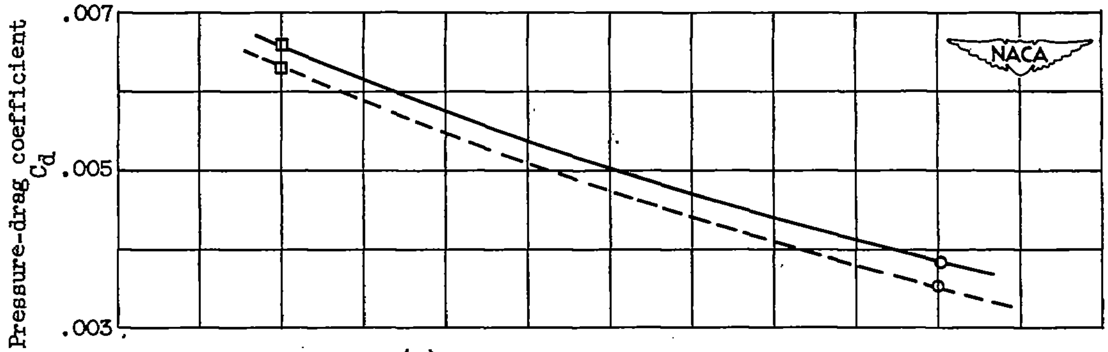
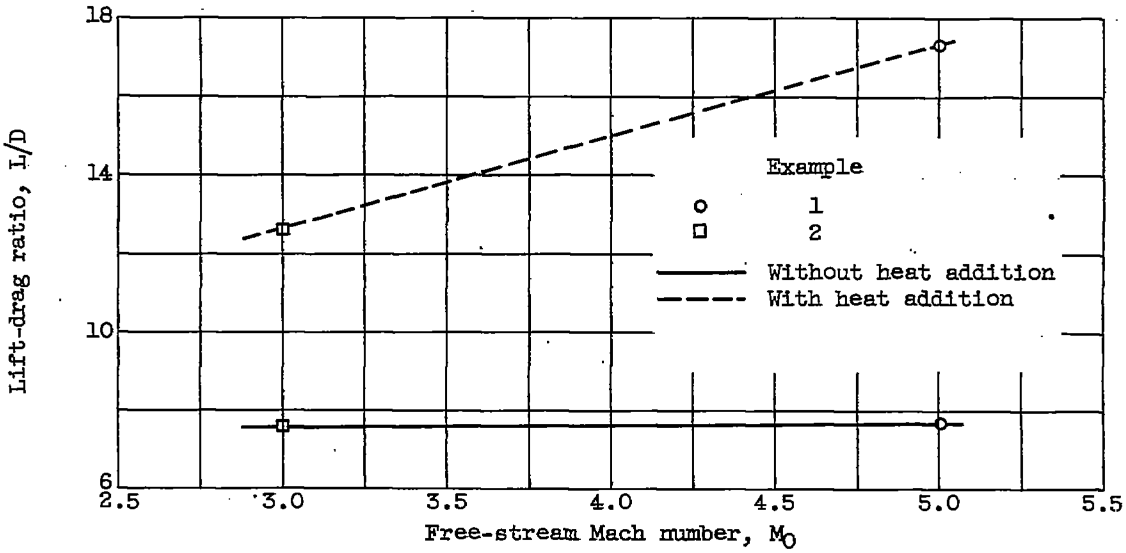


(c) Pressure-drag coefficient.



(d) Lift-drag ratio.

Figure 3. - Concluded. Variation of aerodynamic coefficients with free-stream Mach number.

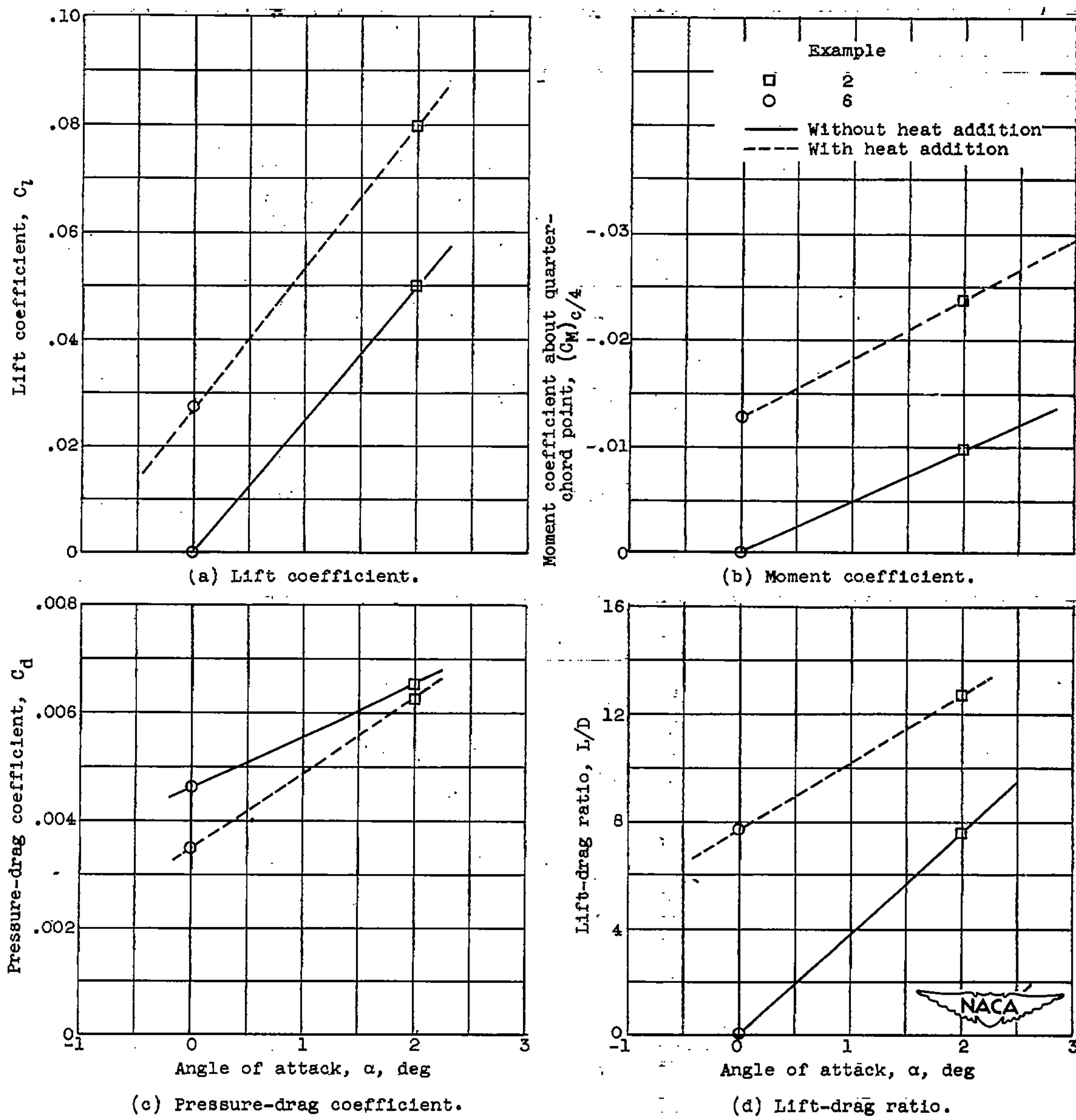


Figure 4. - Variation of aerodynamic coefficients with angle of attack.

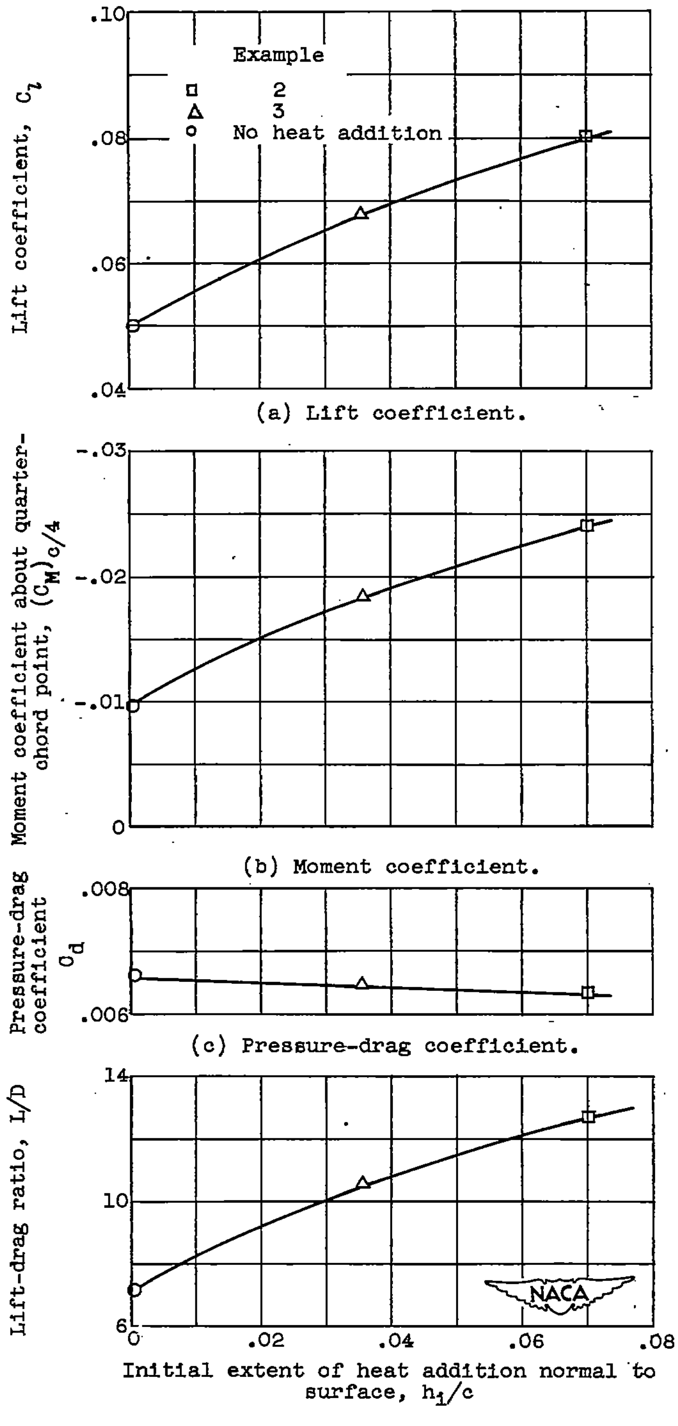
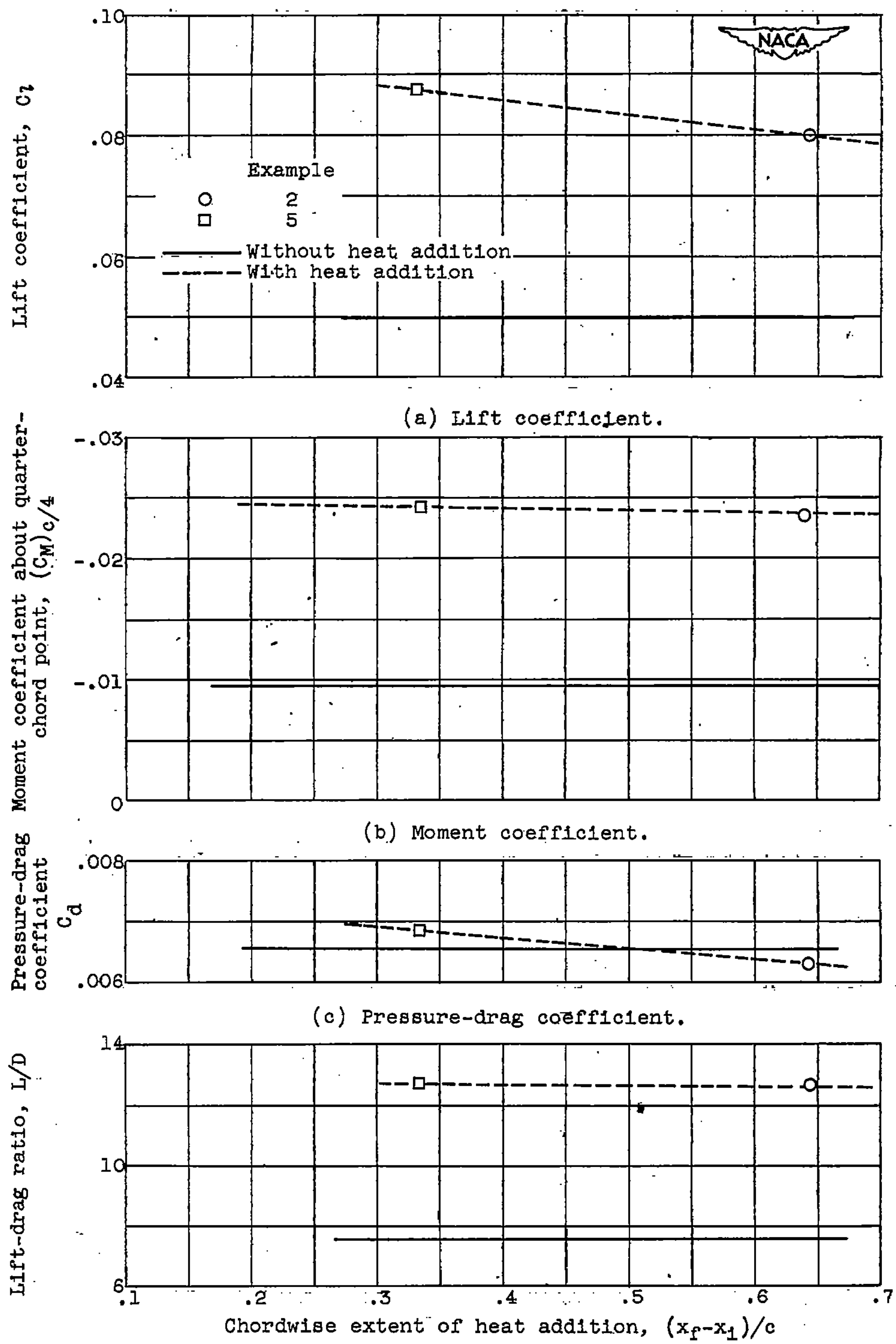
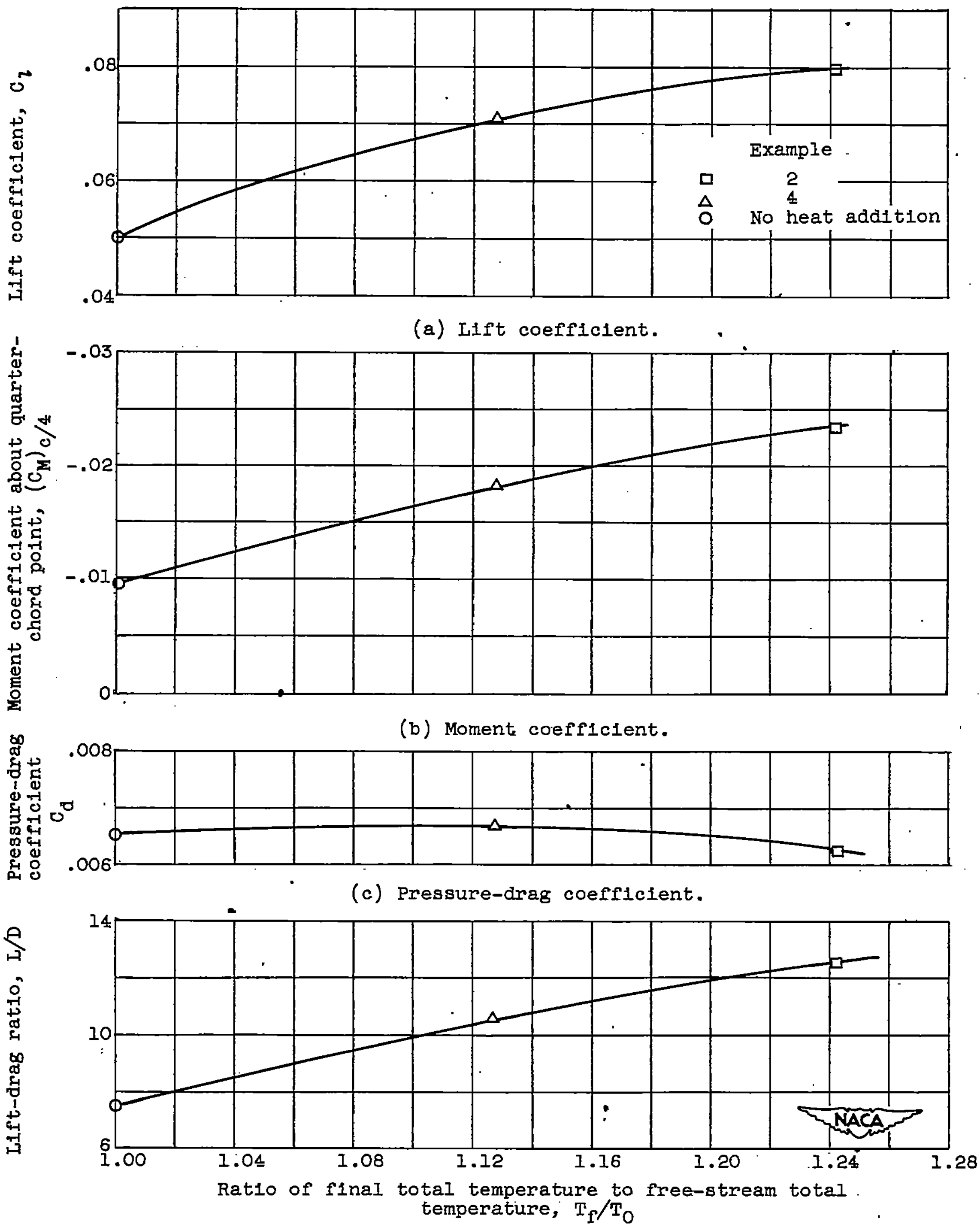


Figure 5. - Variation of aerodynamic coefficients with initial extent of heat addition normal to surface.



(d) Lift-drag ratio.

Figure 6. - Variation of aerodynamic coefficients with chordwise concentration of given quantity of heat addition. x_i/c, 0.356.



(a) Lift coefficient.

(b) Moment coefficient.

(c) Pressure-drag coefficient.

(d) Lift-drag ratio.

Figure 7. - Variation of aerodynamic coefficients with stagnation-temperature ratio T_f/T_0.

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